

Wigner crystals of ions as quantum hard drives

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Atomic systems in regular lattices are intriguing systems for implementing ideas in quantum simulation and information processing. Focusing on laser cooled ions forming Wigner crystals in Penning traps, we find a robust and simple approach to engineering non-trivial 2-body interactions sufficient for universal quantum computation. We then consider extensions of our approach to the fast generation of large cluster states, and a non-local architecture using an asymmetric entanglement generation procedure between a Penning trap system and well-established linear Paul trap designs.

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Quantum information processing using trapped ions has been the focus of theoretical [1] and experimental [2, 3, 4, 5, 6] efforts over the past decade. The coherence times of ions can exceed seconds, while manipulation and entanglement time scales can be as fast as tens of microseconds. So far, approaches to scaling these systems to many ions have met with significant issues, both in linear Paul trap systems, where increasing numbers of ions leads to control difficulties, and in proposed more complex trap arrays, where “shuttling” of quantum information using gate electrodes would allow for a scalable architecture [7]. A possible solution is to separate the processing elements (processor qubits) from the memory [8].

One natural system to consider as a quantum memory is a Wigner crystal of ions in a Penning trap [9]. Such crystals can be robustly formed [10], and are dynamically stable, with tens of thousands of ions in a given trap. In addition, the strength of the Coulomb interaction leads to large separations between individual ions, making individual addressing of ions in such lattices a distinct possibility, in contrast to present control in neutral atom and polar molecule lattices [11, 12].

In this Letter we develop an approach to quantum memory and entanglement generation that takes full advantage of the advances in ion trap technology for building large Wigner crystals of ions in Penning traps. Using a modulated-carrier “push” gate adapted from linear ion trap quantum computing schemes [13, 14, 15, 16], we find a fast but adiabatic method for building small clusters of entanglement which is insensitive to thermal phonons in 2D and 3D Wigner crystals. We take advantage of some of the unique features of Penning traps, such as rotation of the crystal, to provide simplifications in the necessary hardware to implement these ideas in 2D Wigner crystals. We further show that such a quantum memory device can also be used directly for cluster state quantum computation. Our approach follows recent work [17] on performing quantum gates in 2D Wigner crystals. Finally, non-deterministic entanglement generation between distant ions suggests a processor (linear

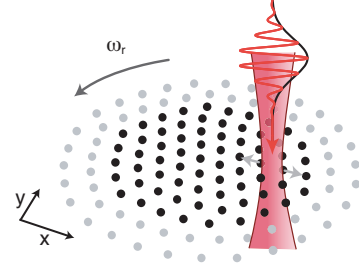


FIG. 1: Two-qubit gate via intensity modulation of a laser addressing a pair of ions in a crystal rotating at frequency ω_r .

Paul trap) and memory (2D Wigner crystal) architecture based upon a quantum register approach [8, 18], where the low photon collection efficiency from ions in the memory is offset by an asymmetric entanglement generation scheme using a weak cavity coupled to ions in the processor [3, 19].

We start by considering a Wigner crystal of ions, rotating in a Penning trap [9] with harmonic confinement with frequencies ω_{xy} (in the lateral directions) and ω_z (in the vertical direction). With characteristic ion spacings $d \sim 10 \mu\text{m}$, tightly focused lasers allow for individual addressing of ions (see Fig. 1). Laser cooling can reduce the temperature $\approx 1 \text{ mK}$, yielding on the order of $10^2 - 10^3$ phonons in the softest (lateral) modes. By using long-lived, metastable states of the ions as a quantum memory, we may neglect memory errors in our discussion. A tightly-focused laser allows for nearest-neighbor phase gates and for single ion operations. Large-scale computation may be considered using either nearest-neighbor couplings or via a variety of quantum communication techniques developed for quantum repeater protocols. When used in conjunction with the deterministic phase gate developed below and local single ion operations (performed, e.g., via Raman transitions), this will suffice for performing large scale quantum algorithms [18] by using the remote CNOT gate [21].

Modulated-carrier gate— A spatially inhomogeneous laser detuned from the appropriate transitions between

internal (qubit) states of an ion (a two-level system with Pauli matrices $\sigma_i^{x,y,z}$) produces a ponderomotive force \vec{f}_i due to the gradient in its intensity. Using an appropriate combination of polarizations and frequencies, in analogy with alkali atoms [20], the sign of the force becomes dependent upon the internal state of the ion, with the associated perturbation to the system:

$$V = \sum_i \left[\vec{x}_i \cdot \vec{f}_i(t) \right] \sigma_i^z \quad (1)$$

where \vec{x}_i is the displacement of ion i away from its equilibrium position. The latter can take place either along the separation between two individual ion microtraps [14] or perpendicularly to the plane of an ion Wigner crystal in a Penning trap [17]. In both schemes, as the ion displacements are coupled (via phonons), such a push leads to an effective $\sigma_i^z \sigma_j^z$ interaction. Adiabaticity is required for vibrational excitations to be absent after the gate. This bounds the clock speed to be lower than the frequency of trapping in the push direction: tight traps are needed for fast, temperature-insensitive operation.

We now introduce a simple variant of the fast-kick “push” gate which allows us to use even the soft (lateral) modes when their temperature is extremely high. Our variant uses slow modulation of a fast, oscillating state-dependent force. The oscillation averages any ion motion to zero over the course of the gate, while the in-phase oscillation of nearby ions leads to a non-trivial phase evolution and the desired quantum gate between ions in the crystal. In addition, as our gate allows for non-trivial oscillation of ion positions in all three spatial dimensions (versus only in the tightly confined direction for the vertical gate), it can work using a single laser beam and in three dimensional crystals.

It is instructive to recall the general description of “push” phase gates when in a complex crystal [15, 17]. We start by rewriting the Hamiltonian of N interacting ions to second order in displacement from the equilibrium positions $H = \sum_K \hbar \omega_K \hat{a}_K^\dagger \hat{a}_K$, using normal-mode coordinates indexed by $K = \{\vec{k}, \lambda\}$ (the wavevector and polarization), $\vec{x}_i = \sum_K M_{iK} \vec{e}_K (\alpha_K / \sqrt{2}) (\hat{a}_K + \hat{a}_K^\dagger)$. The $\alpha_K = \sqrt{\hbar / m \omega_K}$ are the oscillator ground state lengths; the matrix M is orthogonal ($M^t M = M M^t = 1$). The perturbation V can now be written as

$$V = \sum_K \alpha_K f_K(t) (\hat{a}_K^\dagger + \hat{a}_K) / \sqrt{2}, \quad (2)$$

where $f_K(t)$ is the state-dependent force on normal mode K defined via the transform M and Eqn. 1.

The problem factorizes into $3N$ independent, driven oscillators. For scenarios with $\lim_{t \rightarrow \pm\infty} f(t) = 0$, the oscillator evolution is given by the unitary transform $U_K(t) = e^{-i\phi_K(t)} \exp(\beta_K \hat{a}_K^\dagger - \beta_K^* \hat{a}_K)$, where ϕ_K and β_K

satisfy the differential equations [15]

$$\dot{\beta}_K = -i\omega_K \beta_K + i \frac{\alpha_K}{\hbar \sqrt{2}} f_K(t), \quad \dot{\phi}_K = \frac{\alpha_K}{\hbar \sqrt{2}} f_K(t) \mathcal{R}e[\beta_K(t)] \quad (3)$$

which are exact to second order.

We now seek an approach which still maintains no net change in displacement and no dependence of the overall phase on phonon state, but can operate on time scales on the order of ω_K . We add a sinusoidal variation to the force [$f(t) \rightarrow \cos(\nu t) f(t)$]. The carrier frequencies ν must be fast with respect to ω_K ; qualitatively, this averages out any net displacement. If the modulation $f(t)$ is slow as compared to ν (but with no restriction with respect to ω_K), we can perform a similar adiabatic elimination as above, and get a gate with the same desirable properties that can operate non-trivially on arbitrarily “soft” phonon modes at very high temperatures.

For adiabatic elimination with respect to ν , we choose the ansatz $\beta = \beta_+ e^{i\nu t} + \beta_- e^{-i\nu t}$ (subscripts omitted for clarity). Setting $\dot{\beta} = 0$ yields $\beta_\pm = \alpha f(t) / [2\sqrt{2}\hbar(\omega \pm \nu)]$. We find the displacement of a normal mode induced by the gate is proportional to the force applied, and can be made zero independent of initial phonon state by starting and ending with zero force. This eliminates any potential error due to entanglement between phonons and the internal states of the ions.

We now examine the two-body phase induced in this new scenario. The differential equation for phase is now:

$$\dot{\phi} = \frac{\alpha^2}{2\hbar^2} f^2(t) \frac{\omega}{(\omega^2 - \nu^2)} \cos^2(\nu t). \quad (4)$$

Averaging the quickly varying component lets us replace $\cos^2(\nu t)$ with $1/2$. Returning the mode index, K , we find the overall phase accumulated, $\sum_K \phi_K(T)$, for a gate occurring over a time 0 to T does not depend on the phonon initial state. However, the internal states of the ions are affected by the unitary $\exp(-i \sum_{ij} \phi_{ij} \sigma_i^z \sigma_j^z)$ where the two-body phases are given by

$$\phi_{ij} = \sum_\lambda S_{ij}^\lambda \int_0^T (\vec{f}_i(t) \cdot \vec{e}_\lambda) (\vec{f}_j(t) \cdot \vec{e}_\lambda) dt. \quad (5)$$

The pulse-shape independent form factor is

$$S_{ij}^\lambda = - \sum_k \frac{\alpha_{k,\lambda}^2 \omega_{k,\lambda}}{4\hbar^2 (\nu^2 - \omega_{k,\lambda}^2)} M_{ik,\lambda} M_{jk,\lambda} \quad (6)$$

(the polarization vectors \vec{e}_K only depend on λ).

Expanding in inverse powers of the large carrier frequency ν , we note that the first term, $O(\nu^{-2})$, is proportional to $\sum_k M_{ik,\lambda} M_{jk,\lambda} = 0$ (due to the orthogonality of the matrix M). The first non-zero term is $O(\nu^{-4})$. Compared to adiabatic gates, this modulated-carrier push gate is inverted in sign and reduced in phase by a factor $(\omega/\nu)^4/2$. For lateral gates, $\omega \sim \omega_{xy}$ is a characteristic confinement energy for a single ion in the crystal, and for vertical gates, $\omega \sim \omega_z$.

Performance— We performed numerical simulations of the modulated-carrier gate’s performance for finite-size 2D and 3D Wigner crystals ($N = 147$ shown in Fig. 2) to compare to the equivalent adiabatic gate and the proposed vertical gate of Ref. 17. Our simulations minimized the classical energy of the ions in a Penning trap to determine the equilibrium configuration. Then, expanding to second order in displacements from equilibrium, the normal mode coordinates were found. Coriolis forces were neglected; their inclusion does not qualitatively change our results. A gate between the two centermost ions was simulated by computing the displacement β_K and phase ϕ_K for each phonon mode. The fidelity was calculated by considering the overlap of the final state with the desired state, traced over the phonon degrees of freedom and minimized over all possible initial states.

We find that for the same physical parameters, the ratio of forces (i.e., laser power) required for achieving a π phase for the vertical gate and the fast carrier gate goes as $(\omega_z/\nu)^2$, consistent with the ratio between adiabatic and fast carrier gates derived above. Thus, the fast carrier gate requires substantially less laser power for the same conditions with negligible reduction in fidelity. Alternatively, the gate time could be reduced, enhancing the overall performance of quantum information protocols. For specificity, setting $\omega_{xy} = 200$ kHz, $\omega_z = 10$ MHz, and a gate time $\tau = 5$ μ s, we find $\nu = 2.2$ MHz provides $1 - F < 10^{-5}$ with negligible heating. Even smaller errors are found in simulations of the 3D crystal under the same approximations.

A practical limitation occurs due to the spontaneous emission induced by the off-resonant laser interactions. Tight focusing increases the force for the same laser power; thus, using a pair of adjacent, narrow-waist ($\lesssim 2$ μ m) laser beams reduces spontaneous emission and power requirements. For specificity, using a transition with spontaneous emission of $\gamma = 20$ MHz and lasers with peak Rabi frequency of 100 GHz detuned 100 THz from the atomic transition, a laser power of ~ 3 mW per beam is required for our gate, with an induced error of $\lesssim 0.1\%$ per gate.

Quantum cluster state generation— We now consider an approach that takes advantage of the Coulomb interactions in the lattice to create and dynamically extend a cluster state for universal measurement-based computation. Specifically, the goal is to obtain a weighted-graph state $\exp(i \sum_{ij} \sigma_i^z \sigma_j^z \vartheta_{ij}/2) |+\dots+\rangle$, where in the ideal case ϑ_{ij} equals π between nearest neighbors on a square lattice, and zero otherwise. On a triangular lattice like the one available in many-ion Penning traps, this can be achieved if ϑ_{ij} is made to vanish along one side of each lattice cell, and to be π on the other two. The idea is to obtain this via a global $\pi/2$ qubit rotation followed by a push gate acting on all three cell vertices at the same time, possibly with a laser swept at constant velocity through the cell itself, to take advantage of the uniform

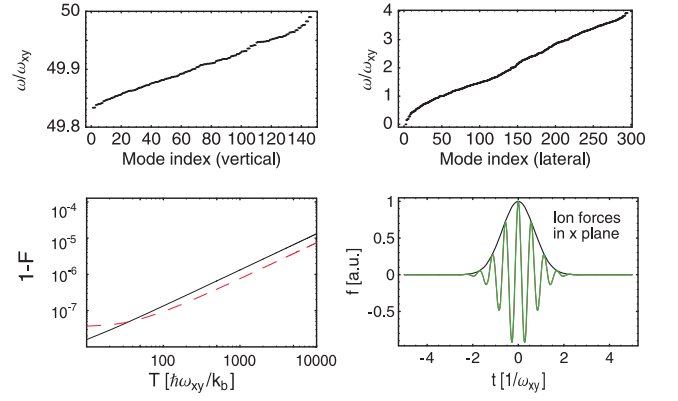


FIG. 2: (top) Phonon spectrum for vertical and lateral phonons for a trap with $N = 147$ ions and $\omega_z/\omega_{xy} = 50$. (bottom) Fidelity versus temperature for the vertical-phonon-mode gate of Ref. [17] (red-dashed line) and for the fast carrier gate with $\nu = 11\omega_{xy}$ between the center-most pair of ions. Left: fast carrier gate’s and vertical gate’s forces on one of the two ions over the gate time; both gates operate in a time $\tau \sim 1/\omega_{xy}$. Anharmonic corrections to the fidelity are not included here. For this choice of parameters, the vertical gate [17] requires 20 times the force (and laser power) of our fast carrier gate to achieve the same final π two-body phase.

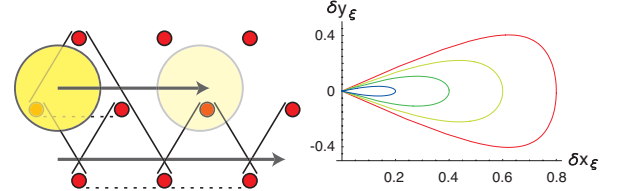


FIG. 3: (color online) a) A laser is swept adiabatically from left to right, leading to a weighted graph state with different phases for solid and dashed lines. b) Laser displacements in the crystal plane needed to obtain a constant sweep velocity at given distances $\xi/R = 0.2, 0.4, 0.6, 0.8$ (from blue to red) from the center, in the rotating crystal’s frame.

circular motion of the lattice.

We start by considering a focused laser beam of waist σ (in units of the lattice length d) adiabatically swept at constant velocity v through the Wigner crystal, along a direction parallel to one of the lattice vectors (Fig. 3), at half the height of a triangular cell. The effect of this sweep is, apart from a global single-qubit rotation, to generate a weighted-graph phase, where ϑ_{ij} takes value $\varepsilon\theta(\omega)$ on the cell side that is parallel to the sweep direction, and $\theta(\omega)$ on the other two sides, with $\varepsilon = e^{-3/(8\sigma^2)}(11 - 8\sigma^2)/(\sigma^2 + 8)$, while

$$\theta(\omega) = \frac{\Omega_0^4}{\omega^2 \Delta^2} \frac{\alpha^4}{d^4} \frac{q^2}{\hbar \epsilon_0 v} \frac{e^{-1/(2\sigma^2)}}{\sqrt{8\pi\sigma}} \left(\frac{1}{\sigma^2} + \frac{1}{8} \right), \quad (7)$$

where $\alpha = \sqrt{\hbar/(m\omega)}$, Ω_0 is the peak Rabi frequency corresponding to the center of the laser beam, Δ is its detuning from the ion’s internal transition, and q is the

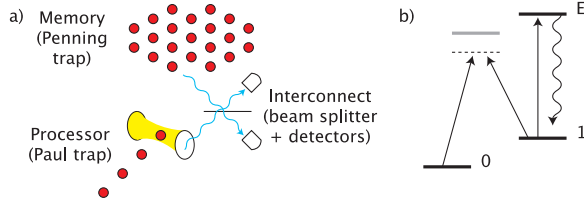


FIG. 4: a) Schematic of a quantum processor (such as a linear Paul trap) coupled via a high finesse cavity to a photodetector system to allow interference with photons from an ion Wigner crystal in a Penning trap nearby. b) Ion level structure

electron charge. Using the fast carrier modulation described above, the semiclassical calculation of Eq. (7) is no longer valid, but the discussion of Eq. (6) shows that the resulting phase is simply $-\theta(\nu)/2$. A cluster state is then obtained by making ε small via an appropriate choice of the laser waist (numerically, $\sigma \lesssim 0.2$), while tuning $\theta(\nu)/2$ to π by adjusting the other experimental parameters such as laser power.

Care needs to be taken to ensure a sweep having a given distance ξ from the trap center and velocity v in the rotating crystal's frame. To this end we apply to the laser, initially focused at a distance R from the center, a displacement $\delta \mathbf{d}_\xi(t) = \{\delta x_\xi(t), \delta y_\xi(t)\}$ of the form

$$\delta x_\xi(t) = [R - \xi / \cos(\omega_r t)] \Theta(\chi - |\omega_r t|), \quad (8)$$

$$\delta y_\xi(t) = \xi [\omega_r t \tan(\chi) / \chi - \tan(\omega_r t)] \Theta(\chi - |\omega_r t|) \quad (9)$$

(see Fig. 3), where $\chi \equiv \arccos(\xi/R)$. Cluster state generation as presented uses a single laser beam, resulting in substantially higher laser power requirements than the two qubit gate with two beams as described above. In particular, for errors per gate $\lesssim 0.1\%$, a detuning of 200 THz and peak Rabi frequency of 4 THz (corresponding to 5 W of laser power) would be required. More complex laser motion could reduce these requirements.

Asymmetric entanglement generation— We conclude with a brief discussion on the implementation of circuit-based computation using entanglement generation and remote CNOTs. We use a quantum processor unit (such as a linear Paul trap) separated from the quantum memory unit (our Wigner-crystal-based quantum hard drive – see Fig. 4), characterized by photon collection efficiencies η and η' respectively. Without loss of generality, we will assume $\eta' > \eta$, as it can be achieved via coupling with high finesse cavities [3, 19]. Our two-click asymmetric entanglement generation procedure starts with an equally weighted superposition $|+\rangle = |0\rangle + |1\rangle$. An optical π pulse produces photons via spontaneous emission at a rate γ from the $|1\rangle \leftrightarrow |E\rangle$ transition. Then the photons are interfered on a beam splitter. Without assuming photon number resolving detectors, the state after one “click” becomes

$$\eta' |\psi_\pm\rangle \langle \psi_\pm| + O(\sqrt{\eta'}) |11\rangle \langle 11|, \quad (10)$$

with $|\psi_\pm\rangle \equiv \sqrt{\eta/\eta'}(|01\rangle \pm |10\rangle)$. To symmetrize the entangled state and simultaneously remove the $|11\rangle$ component, a π pulse between the metastable states ($|1\rangle \leftrightarrow |0\rangle$) followed by repetition of the above protocol results in a pure state $|\Phi^+\rangle = (|01\rangle \pm |10\rangle)/\sqrt{2}$. The overall procedure succeeds with probability $\eta\eta'$, indicating that the time required is $(\gamma\eta\eta')^{-1}$. A standard one-click scheme [22] with excitation probability p takes a time $(\gamma\eta p)^{-1}$, and succeeds with error rate $O(p)$, i.e., the higher fidelity a pair one wishes to generate, the longer it takes. By contrast, in our scheme the fidelity can be high without a further increase in generation time.

Thus, for large-scale computation, a central processor unit with high collection efficiency allows for high-fidelity gates between elements of the “hard drive” memory on a timescale $2/\Gamma\eta\eta'$ (see Ref. 18 for further improvements). For concreteness, we take a radiative decay rate of $\Gamma = (2\pi)10$ MHz, $\eta = 10^{-3}$ (confocal approach with low numerical aperture lens), and desired infidelity $1 - F < 10^{-4}$. Entanglement generation between two such ions would take a time ~ 10 ms or longer; in contrast, for $\eta' \sim 0.1$, using the intermediate quantum processor leads to entanglement generation between processor and both ions in a time of order $100 \mu\text{s}$, comparable to the phase gate operation times already discussed.

This complements the quantum hard drive architecture described above, providing a comprehensive toolbox for universal quantum computation with ion crystals in Penning traps that relies on existing technologies under available experimental conditions.

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